esi Engineering Bulletin
Electro Scientific Industries
7524 S. W. MACADAM AVE. • PORTLAND 19, OREGON

BRIDGES
AND
ACCESSORIES
no. 23

NOVEMBER 1962
REPLACES JANUARY 1961

A NEW APPROACH TO BRIDGE SENSITIVITY

by
Jack Riley

Presented as Paper Number 23-SL-61
on January 18, 1961 at the

INSTRUMENT SOCIETY of AMERICA

WINTER INSTRUMENT-AUTOMATION CONFERENCE AND EXHIBIT
ST. LOUIS, MISSOURI • JANUARY 17-19, 1961

© Copyright 1962 by Electro Scientific Industries, Inc.
LITHO IN U.S.A.

A NEW APPROACH TO BRIDGE SENSITIVITY

Ever since Christie invented the Wheatstone bridge in 1833, people have been concerned with bridge sensitivity. In 1862, Thomson released a paper describing his Kelvin double bridge (1). The sensitivity analysis in his paper is believed to be the first published. He concluded that the heating effect of the test current was the limiting factor. This conclusion was overlooked for the next three decades mostly because of the low quality of the batteries available for powering bridge circuits. Schwindler, in 1866, Heaviside in 1873, and Gray in 1881, all attempted bridge sensitivity analysis. The second and third editions of Maxwell's "Electricity and Magnetism," published in 1892, include a section on bridge sensitivity. The modern approach to bridge sensitivity analysis is given by Wenner (1) in Volume 25 of the "Journal of Research of the National Bureau of Standards."

Most of the writings about bridge sensitivity finish with an impressive algebraic expression which must be evaluated for each possible setting of the bridge. This paper presents a graphical analysis offering several advantages. The calculations for a large number of bridge settings can be performed rapidly. The behavior of the bridge and its generator are separated from the performance of the detector. The suitability of various detectors can be checked in a simple manner. Sensitivity curves for different bridge and detector combinations can be constructed easily, or sensitivity can be deduced from the bridge and detector plots directly.

The graphical technique for analyzing a Wheatstone bridge is given. A typical circuit is used as an example and the performance for a particular measurement is followed through all of the steps. The performance curves for a commercially available bridge are given to show the practical application of the technique.

WHAT IS SENSITIVITY

A definition of sensitivity is needed before a bridge can be analyzed. Sensitivity is defined as the ratio of the smallest discernible change of the measured quantity to the total value of that quantity. If a 1% change in the resistance being measured produces a barely perceptible movement of the null indicator, the sensitivity is 1%.

GRAPH PAPER

Special graph paper simplifies the graphical analysis. The coordinates of this paper are log power versus log resistance, with contours of equal voltage and equal current drawn on the diagonals, as shown in Figure 1. With this paper Ohm's law and the power equation are solved simultaneously. A point plotted on this graph paper represents the resistance value of a resistor, the voltage across the resistor, the current through the resistor, and the power dissipated by the resistor.

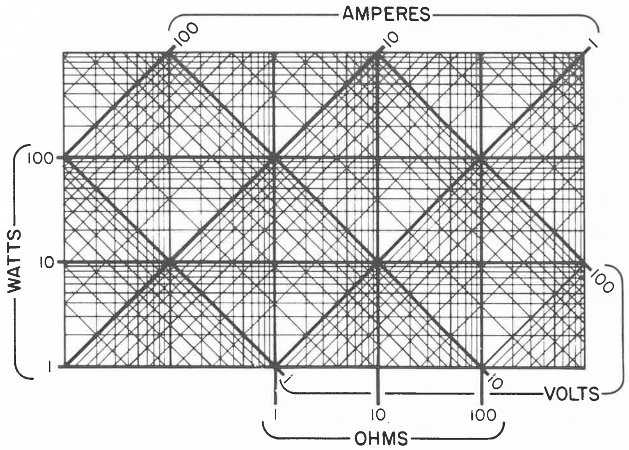


FIGURE 1. GRAPH PAPER

ANALYZING A BRIDGE

The sensitivity analysis technique can be illustrated by applying it to the bridge circuit shown in Figure 2. The components for this bridge, the generator which will drive it, and the detector which is used as a null indicator are chosen, then the analysis is made. First the bridge output resistance and open-circuit voltage are found. Their effect on a detector is then studied.

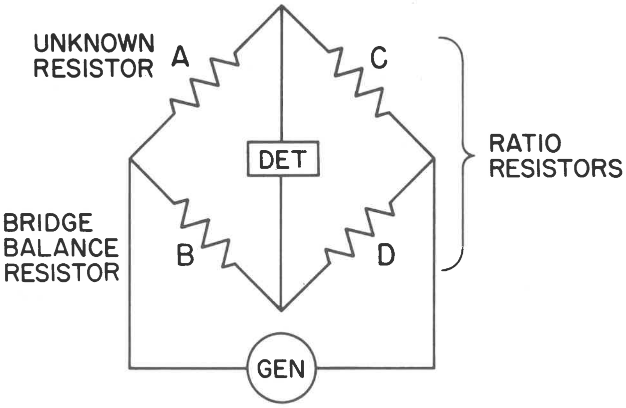


FIGURE 2. BRIDGE CIRCUIT

THE GENERATOR

It is particularly informative to plot the equivalent Thevenin or Norton generator performance on the graph paper shown in Figure 1. For either type of equivalent circuit (and for most practical circuits) the open circuit voltage, the short circuit current and the source resistance all intersect at a single point. This can be called the "source describing point". This single point on the graph paper as shown in Figure 3 serves to completely describe the output of the generator in a manner which can be later used to find the amount of voltage or current it will actually supply to a bridge.

In Figure 3 the power supplied to the load is plotted versus the load resistance. If the load resistance is low, a constant current approximately equal to the short-circuit current is supplied. If the load resistance is high, a constant voltage approximately equal to the open-circuit voltage is supplied. The curve can be predicted on the basis of either the Thevenin equivalent generator consisting of a zero-impedance voltage source in series with a fixed resistor, or the Norton equivalent circuit consisting of an infinite impedance current source feeding a fixed internal resistance and load resistance in parallel. The load curve can be plotted from either form of equation (1).

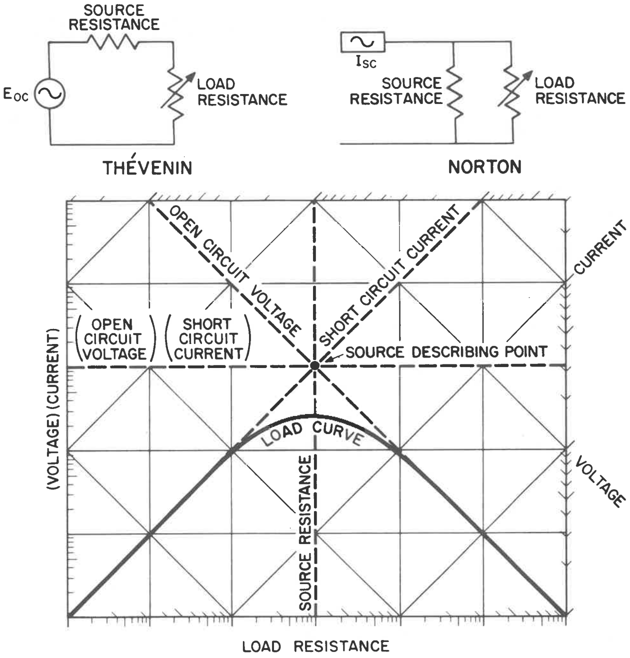
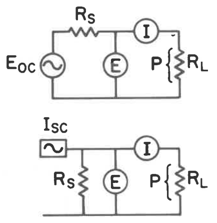


FIGURE 3. SOURCE REPRESENTATION

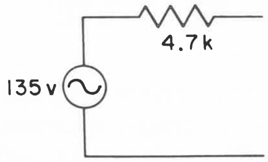


$$P = (E_{OC})^2 \frac{R_L}{(R_L + R_S)^2}$$

$$P = (I_{SC} R_S)^2 \frac{R_L}{(R_L + R_S)^2} \qquad (1)$$

The open-circuit voltage and short-circuit current will intersect at the value of internal resistance. The power at this intersection point represents the maximum amount of power which can be dissipated in the internal resistance of the generator. This intersection point is enough to completely describe a given generator. The curve is always the same shape and can be drawn with a template once the intersection of open circuit voltage and short-circuit current is found.

The generator chosen for the bridge shown in Figure 2 consists of three 45-volt batteries in series with a protective resistor of 4.7k. The performance of this generator is shown in Figure 4.



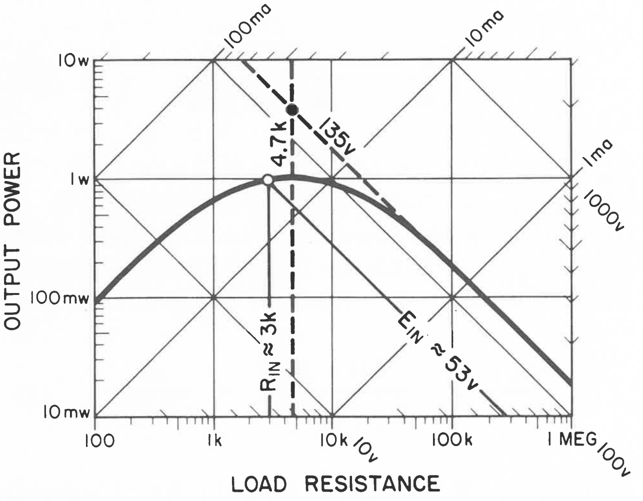


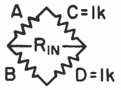
FIGURE 4. GENERATOR PERFORMANCE

When this generator is shorted, short-circuit current of about 30 ma will flow through the protective resistor causing it to dissipate about four watts. The maximum output power from the generator is approximately one watt. It cannot damage any bridge or detector components which are capable of dissipating one watt of power. This form of power protection can often be used to advantage.

If the bridge of Figure 2 is measuring a five-kilohm resistor and if resistors C and D are each one kilohm, the bridge will be a three-kilohm load. The generator will supply about 53 volts to this load as shown in Figure 4.

INPUT RESISTANCES

The bridge input resistance is needed for all values of unknown resistance to be measured. The input resistance is found by paralleling resistors A and C in series with B and D in series. At balance, resistance in the detector circuit has no effect. Equation (2) gives the input resistance.



$$R_{IN} = D\frac{(A+C)}{(D+C)} \qquad (2)$$

The curve for the input resistance also has a characteristic shape and can be drawn with a template. Two calculated values will locate the template so that the complete range of values can be plotted. If A is low, B is also low so the input will look like resistors C and D in parallel as shown in equation (3).

$$R_{IN} \xrightarrow[A,B \ll C,D]{} C\left(\frac{1}{1+\frac{C}{D}}\right) \qquad (3)$$

If A is high, the input resistance will approach the value in equation (4).

$$R_{IN} \xrightarrow[A \gg C]{} A\left(\frac{1}{1+\frac{C}{D}}\right) \qquad (4)$$

These two asymptotes will intersect when A = C. The resulting curve is plotted in Figure 5.

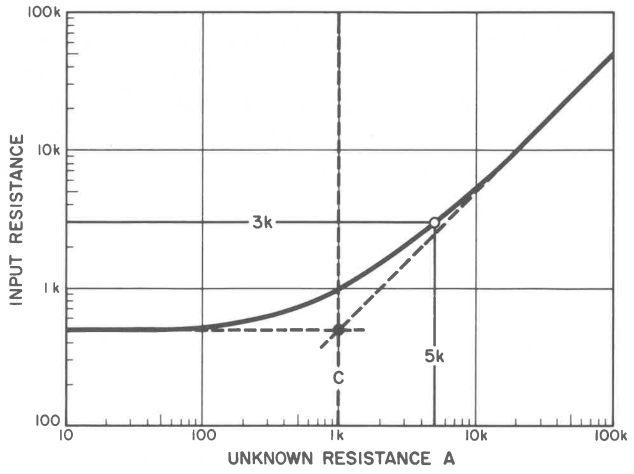


FIGURE 5. INPUT RESISTANCE

INPUT VOLTAGE

The bridge input voltage can be found from the generator performance curve (Figure 4) and the input-resistance curve (Figure 5). The results are plotted in Figure 6. A few points are usually adequate for plotting the entire range of values.

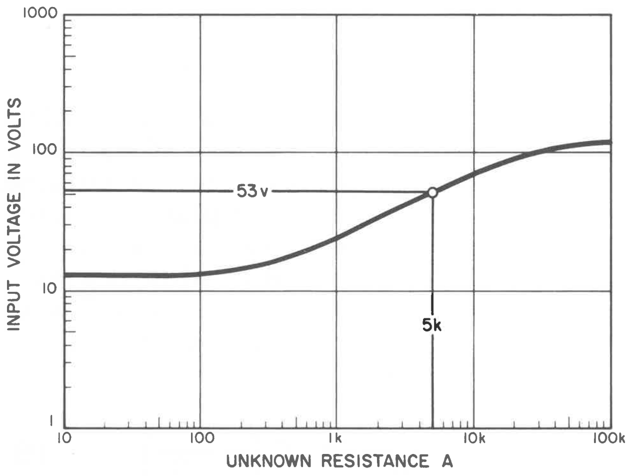


FIGURE 6. INPUT VOLTAGE

TRANSFER FUNCTION

A transfer function can be calculated to relate the open circuit voltage at the detector terminals to the input voltage. When the bridge is balanced, there will be no voltage at the output terminals, so the bridge must be unbalanced by an amount Δ to find a signal to detect. A bridge circuit can be considered as two voltage dividers. At balance both dividers are at the same setting so that their output voltages are equal. Before the bridge is unbalanced, both of the dividers are set so that the voltage E_A across resistor A (the same voltage is also across resistor B) is related to the input voltage by equation (5). When the bridge is unbalanced by

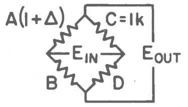
$$\frac{E_A}{E_{IN}} = \frac{A}{A+C} \tag{5}$$

changing the unknown resistor A to A(1 + Δ) the voltage across resistor A changes to E'_A which is related to the input voltage by equation (6). The voltage E_A across resistor B does not change, so the difference between it and E'_A is the open-circuit output voltage E_{OUT} which is found from equation (7). Factoring equation (7) gives equation (8). When Δ is small, as it usually is, equation (8) simplifies to equation (9) which is used for most actual calculations. The plot of equation (9) has

$$\frac{E'_A}{E_{IN}} = \frac{A(1+\Delta)}{A(1+\Delta)+C} \tag{6}$$

$$\frac{E_{OUT}}{E_{IN}} = \frac{A(1+\Delta)}{A(1+\Delta)+C} - \frac{A}{A+C} \tag{7}$$

$$\frac{E_{OUT}}{E_{IN}} = \frac{AC\Delta}{(A+C)^2\left(1+\frac{A\Delta}{A+C}\right)} \tag{8}$$



$$\frac{E_{OUT}}{E_{IN}\Delta} \xrightarrow[(\Delta\to 0)]{} \frac{AC}{(A+C)^2} \tag{9}$$

the same shape as the power curve so that the same template can be used for drawing it. The characteristic point for drawing the curve will have an ordinate of one and an abscissa of the resistance of C. Figure 7 is a plot of the transfer function for the bridge of Figure 2 if C is one kilohm.

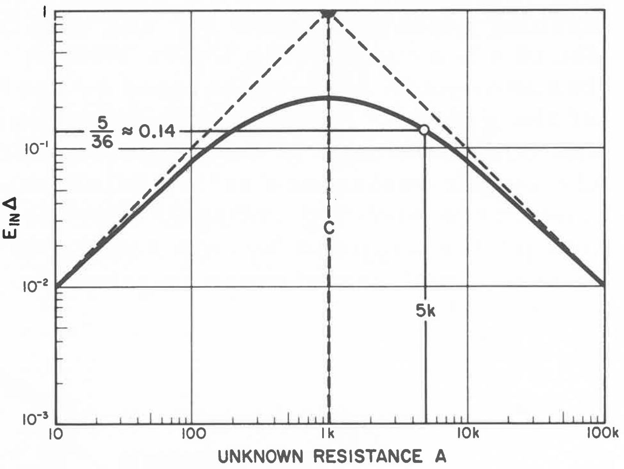


FIGURE 7. TRANSFER FUNCTION

UNBALANCE Δ

The amount of unbalance Δ is a matter of choice. To find sensitivity a fixed value is assigned to Δ and the detector response is found. From this the sensitivity can be calculated. Bridge accuracy or resolution values can be assigned to Δ to find the usefulness of a detector. For the bridge being analyzed Δ is chosen as the specified accuracy and is given by equation (10).

$$\Delta = \pm(1\% + 1 \text{ DIAL DIVISION}) \qquad (10)$$

Δ is plotted in Figure 8.

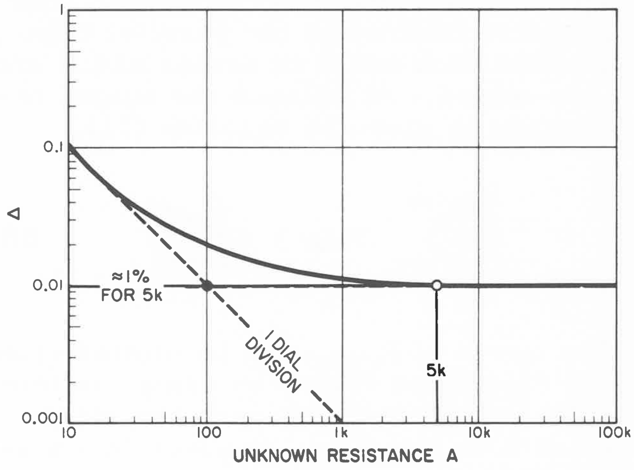


FIGURE 8. VALUES OF Δ

OUTPUT VOLTAGE

From Figures 6, 7 and 8 the output voltage can be found. Figure 9 is the plot of output voltage for the bridge of Figure 2. Both the complete curve for 1% unbalance and the curve for the combined 1% plus one dial division are shown.

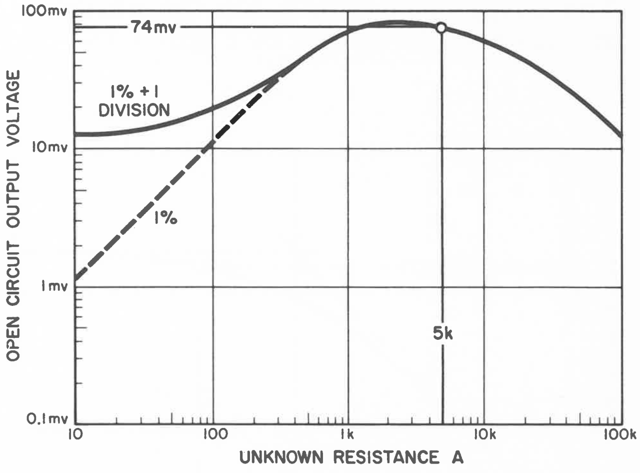
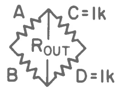


FIGURE 9. OUTPUT VOLTAGE

OUTPUT RESISTANCE

To complete the equivalent-generator circuit for the bridge output the output resistance must be found. The output resistance is the parallel combination of A and B in series and C and D in series. At balance the output resistance is given by equation (11).



$$R_{OUT} = A\frac{(D+C)}{(A+C)} \qquad (11)$$

The curve of R_{OUT} can be plotted from two calculated values by using the template that plotted the input resistance. When A is low R_{OUT} is given by equation (12).

$$R_{OUT} \xrightarrow[A \ll C]{} A\left(1+\frac{D}{C}\right) \qquad (12)$$

When A is high the resistance becomes C and D in series as shown in equation (13).

$$R_{OUT} \xrightarrow[A \gg C]{} C\left(1+\frac{D}{C}\right) \qquad (13)$$

The asymptotes intersect at A = C. The output resistance for the bridge of Figure 2 is shown in Figure 10.

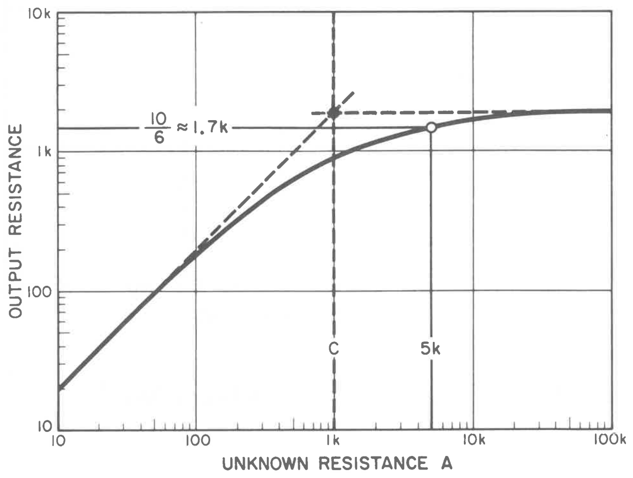


FIGURE 10. OUTPUT RESISTANCE

BRIDGE OUTPUT EQUIVALENT GENERATOR

The output voltage (Figure 9) and output resistance (Figure 10) can be combined to determine the output that the bridge will deliver to any load resistance. The output voltage and output resistance meet at the source describing point of Figure 3. The values found when measuring a five kilohm resistor have been illustrated in each of the graphs. With this measurement the output voltage is 74 millivolts and the output resistance is 1.7 kilohms. The curve showing voltage, current and power supplied by this source to various load resistances is given in Figure 11.

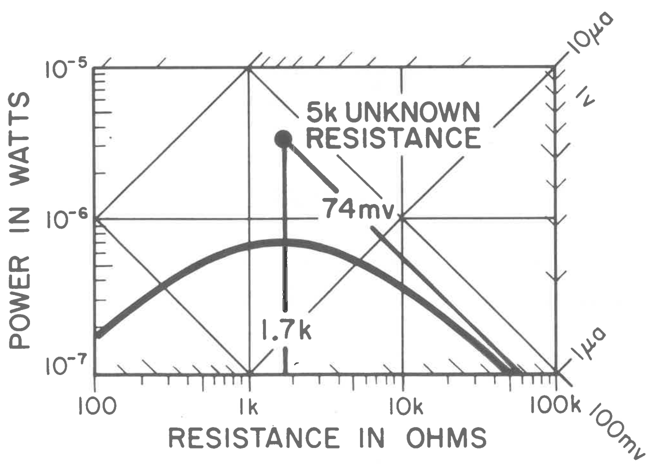


FIGURE 11. BRIDGE EQUIVALENT GENERATOR FOR ONE-PERCENT UNBALANCE WHEN MEASURING A 5k RESISTOR

Similar curves for each value of unknown resistance could be drawn. They would cover most of the paper and therefore be useless. Fortunately a plot of only the source describing points contains all of the necessary information. This graph is made by plotting the intersection points of the output voltage and output resistance for the values of unknown resistance of interest. Figure 12 is the result

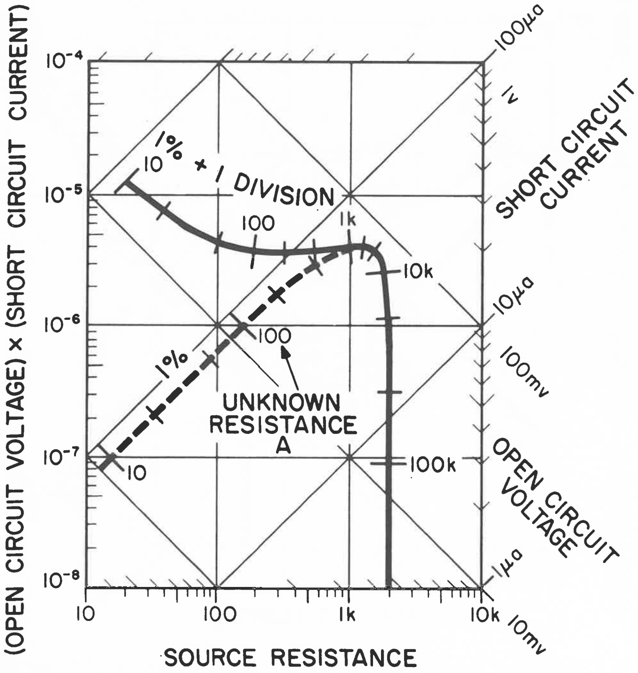


FIGURE 12. BRIDGE EQUIVALENT GENERATORS FOR ALL VALUES OF UNKNOWN RESISTANCE

The solid curve is for a Δ of ±(1% + 1 dial division). The dotted curve is for a Δ of ±1% only.

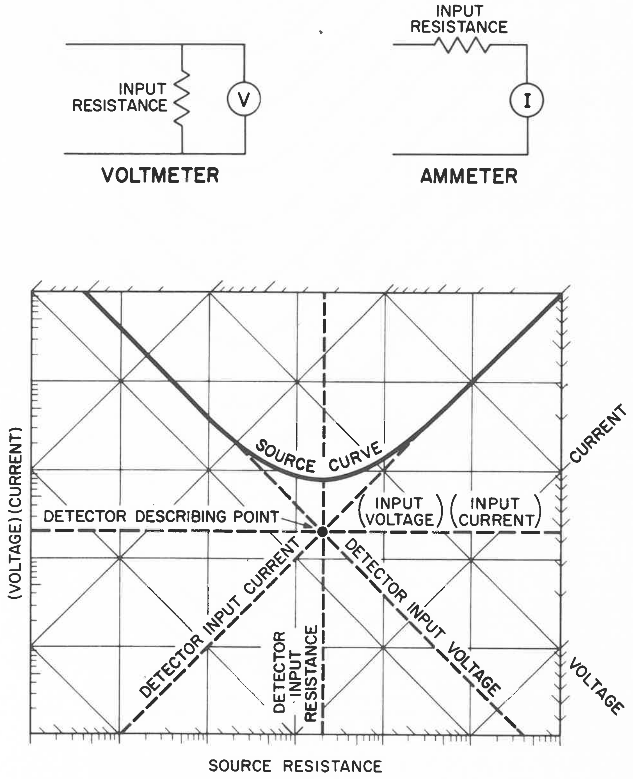


FIGURE 13. DETECTOR REPRESENTATION

THE DETECTOR

It is also possible, as shown in Figure 13, to describe one deflection of the detector by a point. The detector describing point represents its input voltage, current, and resistance for one deflection. A curve can be drawn which represents the locus of generator describing points which would produce this specified detector indication. The curve has the same shape as the generator curve, but it is inverted. If a bridge source describing point lies anywhere on the curve the detector will produce the deflection represented by the detector describing point for the curve. The detector curve can be found experimentally (or calculated) by finding which combinations of open circuit source voltage (or short circuit source current) and source resistance will give the indicated deflection of the detector. Different amounts of indication are represented by different describing points and their corresponding curves as shown in Figure 14.

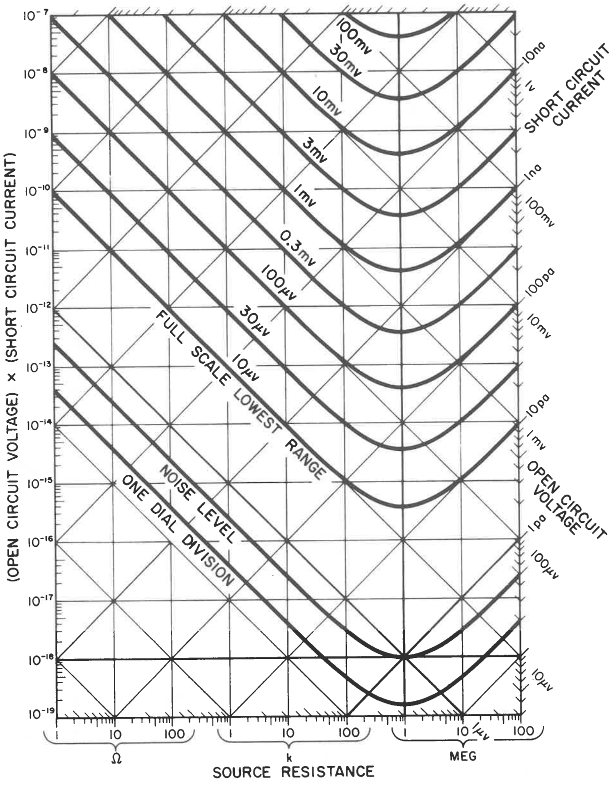


FIGURE 14. DETECTOR CURVES

BRIDGE PERFORMANCE

A generator describing point, such as those plotted in Figure 12, which is above the detector curve will produce greater deflection than that for which the curve was drawn. A generator point below the curve will not produce the indicated deflection. If the detector curve is drawn for minimum detectable deflection (noise level, smallest observable movement, etc.) it will separate generator describing points into those which can and cannot be detected. For the bridge of Figure 2 a detector with one-kilohm input resistance and a minimum observable deflection of ten millivolts was chosen. In Figure 15 this detector plot has been added to Figure 12. The sensitivity is sufficient to see rated accuracy up to about 40 kilohms of unknown resistance. If the bridge controls had sufficient resolution the detector could find one percent differences of resistors down to 100 ohms.

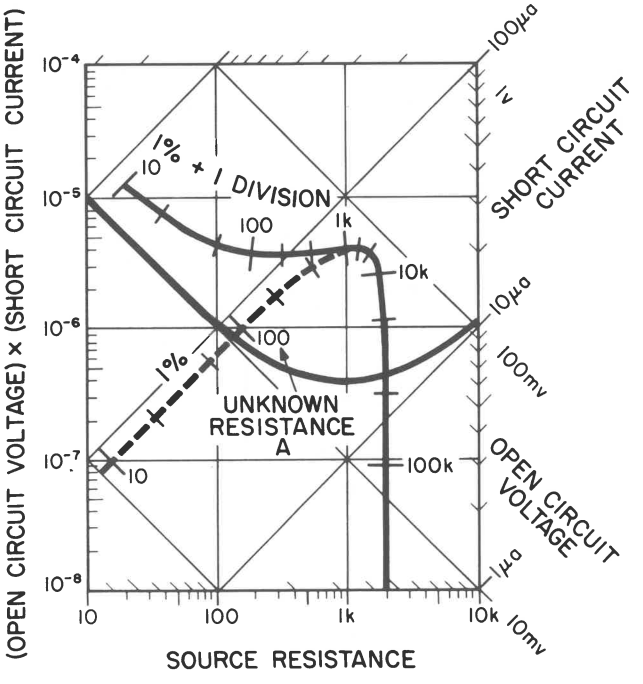


FIGURE 15. BRIDGE AND DETECTOR PERFORMANCE

SENSITIVITY CURVE

The graph provides not only qualitative data, but quantitative data as well. The bridge output voltage is almost directly proportional to Δ. The difference between the value of Δ which was chosen and the value of Δ which would supply minimum detectable signal can be found by vertical measure-

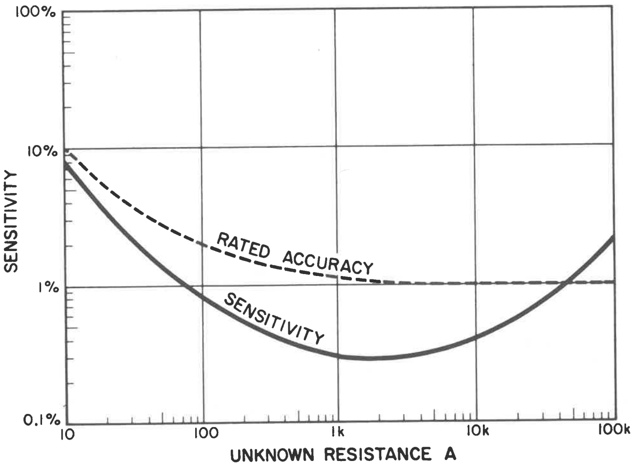


FIGURE 16. BRIDGE SENSITIVITY

ment using the voltage scale. By using this technique, the bridge sensitivity curve of Figure 16 was drawn.

The bridge sensitivity and the bridge specified accuracy are shown in similar units. Although Figure 16 is a more conventional method of presenting the data, the same information is available from Figure 15. The performance of various detectors can be demonstrated more easily by using Figure 15. To predict the performance of a new detector, one need only know its input resistance and its minimum detectable signal in terms of either current or voltage. From these data its curve can be drawn with the template.

DETECTOR COMPARISON

The type of detector curve shown in Figure 13 can also be used to compare one detector with another as shown in Figure 17. Here a rather typical collection of both ac and dc detectors is shown with their maximum sensitivity curves superimposed. It can be seen that some detectors are more sensitive for low impedance sources; others for high impedance sources. A set of describing point curves such as Figure 12, for a particular bridge or instrument can be laid over this set of curves to see which detectors have enough sensitivity for all bridge settings of interest at the required accuracy.

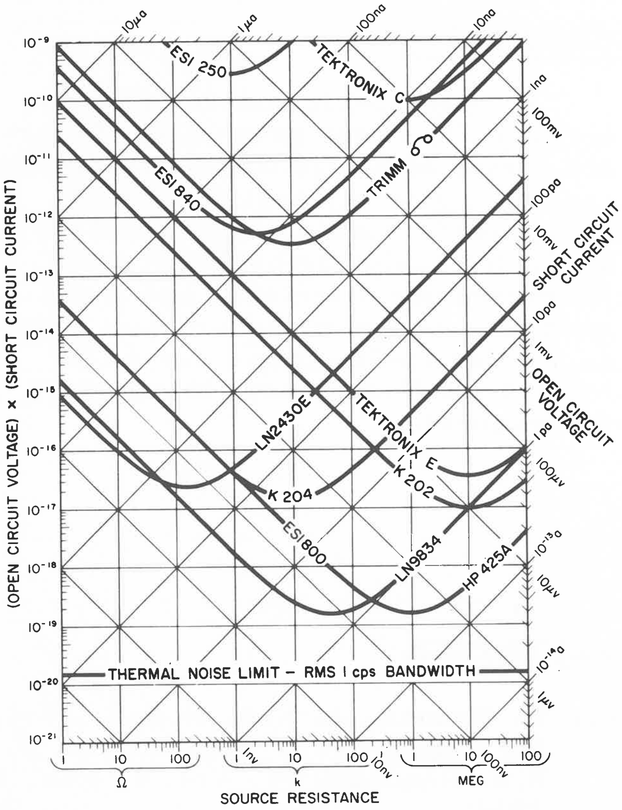


FIGURE 17. DETECTOR COMPARISON

CRITICAL DAMPING

If a low-resistance source is connected to a galvanometer the pointer movement will be extremely sluggish. If the source resistance is too high the needle will swing back and forth a long time without settling down to a final indication. To avoid these problems the source resistance can be kept constant at the critical damping value by adding series or shunt resistance. The constant source resistance requirement modifies the detector curves as shown in Figure 18.

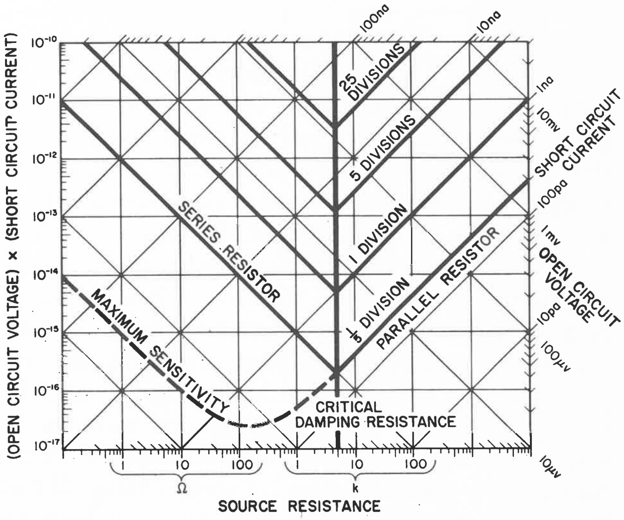


FIGURE 18. GALVANOMETER WITH 5k CRITICAL DAMPING

The dotted line at the bottom is the signal which would produce a minimum visible deflection of one fifth dial division without the extra series damping resistance.

PERFORMANCE OF A MULTI-RANGE BRIDGE

An ESI Model 291 A Resistance Bridge Circuit has been analyzed to show the many useful aspects of the bridge-detector plots. The resulting curves are shown in Figure 19.

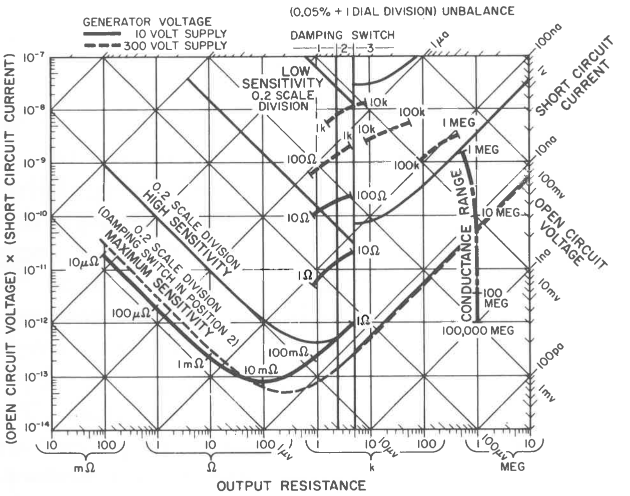


FIGURE 19. BRIDGE PERFORMANCE

The bridge curves are for a ±(0.05% + 1 dial division) unbalance. The solid curves are for a 10-volt generator; the dotted curves represent the bridge performance with a 300-volt supply. Measurements to 1 megohm are with a resistance circuit. From 1 megohm to 100,000 megohms a conductance circuit is used. This bridge has a light-beam galvanometer. Three different meter damping conditions are provided. Optimum damping is obtained by placing a resistance in series with the detector when the supply source has a low resistance, connecting directly to the meter when the source has the proper damping resistance and placing a resistance in parallel when the source has a high resistance. There are three sensitivity conditions and these, combined with the three damping conditions, result in nine combinations of detector performance. The resulting curves for 1/5 scale division on the galvanometer are drawn only for the ranges for which they are applicable. The optimum detector control settings and the measurement sensitivity can be determined from the curves.

For example, measuring a 10-ohm resistor using the 10-ohm to 100-ohm range with the damping switch in position 1 about 15 times as much voltage will be available as is needed for a 0.2 scale division deflection. By changing to damping position 2 some 40 times as much voltage will be available as is needed for 0.2 scale divisions, but the meter will be very sluggish. The center sensitivity position would yield only about 70% of the 0.2 scale division for a 0.05% unbalance. It would require about 0.07% unbalance to yield 0.2 scale division.

CONCLUSION

A bridge-performance analysis technique has been presented. Separate curves of bridge output and detector capability give a visual concept of bridge performance. The technique is useful for analyzing bridge circuits, for comparing detectors, and it can even be extended to such distant fields as plotting speed-torque curves for motors and flux-ampere turn curves for magnets.

REFERENCE

(1) F. Wenner, J. Res. N.B.S., Vol. 25, August 1940. Reprinted in N. B.S. Handbook 77, Vol. 1, Feb. 1, 1961.